The Radar Information Channel and System Uncertainty

John A. Malas
Sensors Directorate, Air Force Research Lab
2241 Avionics Circle, 45433, WPAFB, Ohio
WPAFB, Ohio 45433
John.malas@wpafb.af.mil

John A. Cortese
MIT Lincoln Laboratory
Lexington, MA, 02420-9108
jcort@ll.mit.edu

***Abstract*—The radar information channel is developed as a theoretical model for the study of uncertainty within the design, development, and research of radar signature exploitation systems. Information measures are developed which characterize sources of uncertainty and propagate the associated impacts to system performance. Sources of uncertainty are studied to form an information loss budget for trading component design options against overall system performance.**

I. Introduction

The ability to perform radar system component trade studies in an efficient and meaningful way requires that we study the function of any one component in the context of the system as a whole. The integration science that works at the seams between components is critical to the research of emerging signature exploitation technology. System components include hardware designs within the antenna, receiver, analog-to-digital converter, and signal processor areas as well as software designs to measurement functions, transmit waveforms, digital sampling approaches, and signal processing techniques. These radar systems also include a new list of significant design components that are related to the task of target signature measurement, feature processing, and decision algorithm performance. The use of existing systems theory prototypes such as the radar range equation are useful in studying target visibility, yet fall short in their ability to fully characterize the flow of information through the radar system as it relates to a desired specific exploitation capability. The resulting expanded trade space requires new systems theory models that inherently manage robustness in the face of the complexities involved in achieving optimal component design. The relative success of these efforts will largely depend on our ability to study the performance of modular systems under the effects of various sources of uncertainty. Thus theory models should propagate the effects of these uncertainly sources acting on individual components within the system to the predicted *system* performance measures. Information theory prototypes enable the study of individual components as they relate to system performance within the decision rule subspace. It is this inherent benefit that distinguishes an information theoretic approach over traditional statistical pattern recognition methods.

Woodward and Davies [1] and Woodward [2] were the first to apply the information theoretic approach to the analysis of radar, soon after the appearance of Shannon's original work [3] on information theory. More recently Bell [4] has suggested the use of an information theoretic approach to the design of radar waveforms. Dr. Bell formulated and obtained a solution to the problem of designing a waveform that maximized the mutual information (MI) between the target impulse response (viewed as a random process) and the received signal. Recently, Leshem et al. [5] extended Bell's work to the case of multiple extended targets. Sowelam and Tewfik [6] also used waveform design in conjunction with the Kullback-Liebler [7] criterion to distinguish between different target classes. Briles [8] applied rate distortion theory to analyze the impulse radar for use in target identification design and performance prediction. Home and Malvern [9] introduce a high level theoretical framework to calculate the information conveyed by the image of a target based on pixel values relative to the modeled fluctuations of these values. Principe, Xu, Zhao, and Fisher [10] present a framework for learning based on information theoretic criteria. Methods such as the maximum likelihood test have been used to evaluate radar signature processes for target classification performance as in the work by O'Sullivan et al [11]. This framework proposes several approximations to the Kulback-Leibler divergence that can be used to estimate statistical distances compatible with pattern matching algorithms. Malas and Pasala [12] introduce the use of MI as a similarity measure for use in radar signature database validation. Recently, there has been interest in radars with an architecture referred to as the MIMO (Multiple Input Multiple Output) radar [13] – [16]. It is the information theoretic approach that unifies the analysis of these radar systems.

In varying degrees the body of existing referenced work has (in one form or another) presented the radar system in terms of a Markov Chain within a channel configuration and

978-1-4244-5813-4/10/$26.00 ©2010 IEEE

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **MAY 2010**	2. REPORT TYPE	3. DATES COVERED **00-00-2010 to 00-00-2010**
4. TITLE AND SUBTITLE **The Radar Information Channel and System Uncertainty**		5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)		5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Air Force Research Lab,Sensors Directorate,2241 Avionics Circle,Wright Patterson AFB,OH,45433**		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR'S ACRONYM(S) 11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES
See also ADM002322. Presented at the 2010 IEEE International Radar Conference (9th) Held in Arlington, Virginia on 10-14 May 2010. Sponsored in part by the Navy.

14. ABSTRACT
The radar information channel is developed as a theoretical model for the study of uncertainty within the design development, and research of radar signature exploitation systems. Information measures are developed which characterize sources of uncertainty and propagate the associated impacts to system performance. Sources of uncertainty are studied to form an information loss budget for trading component design options against overall system performance.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**6**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

characterized the information flow within from source and sink. Tishby [17] has developed the information bottleneck approach, wherein rate-distortion theory, the Data Processing Theorem, and compression play major roles. The max-flow min-cut application to the channel problem has been studied to understand the relationship of capacity to information flow. A significant contribution of the work provided herein is the development and demonstration of a systems theory model for the study of the effects of uncertainty on the information flow within the various *components* of the sensor system. The propagation of component level uncertainty to the system performance measure is achieved.

Modern sensing systems produce signature measurements of exploitable physical processes (signal) that when combined with the effects of system uncertainties (Table I) result in an altered signature subspace or distorted decision rule. These effects limit the exploitation of physics-based features and result in a loss in information that can be extracted from signature measurements. Knowledge of the composition and relative weighting of the uncertainty sources *within* the system may allow one to efficiently minimize the overall loss in the information flow while trading costs associated with component design.

TABLE I. RADAR SYSTEM UNCERTAINTY SOURCES

Uncertainty Core Area	***Uncertainty Subcategory***		
1. Signature Measurement[a]	Phase	Amplitude	Thermal Noise
2. Object Tracking & Motion	Object Range, Velocity, & Aspect Estimates	Object Articulation	Intra-measurement Motion
3. Interference	Clutter	RF Interference	Jamming
4. Decision Rule Training Limitations	Data Sparseness	Parameter Variation	Obscuration

a. Traditional Measurement Uncertainty

It is important to contrast the proposed concept of uncertainty with several terms generally used by the measurement community [18]. Accuracy refers to the agreement between a measurement and the true or correct value. Precision refers to the repeatability of a measurement. Error refers to the disagreement between a measurement and the true or accepted value. The uncertainty in a stated measurement is the interval of confidence around the measured value such that the measured value is expected not to lie outside this stated interval. The use of the term "uncertainty" usually implies that the true value may not be known and can be stated along with a probability. Given the nature of the sources of uncertainty identified within this body of work, the condition of "no known truth" is highly relevant. However, uncertainty as defined by the sensor measurement community may not be sufficient to address the full range of issues under study within a radar exploitation system.

II. APPROACH

An information theoretic model is presented to quantify and study the flow of information through the radar system. Analysis of key component design choices including sensor (bandwidth, dynamic range, and signal-to-noise ratio), classifier algorithm training, and algorithm feature design are performed to study sensitivities to information loss due to various uncertainty sources. Insight into the relative effects of component information loss within the system will provide insight as to how to proceed with future studies of uncertainty sources listed in Table I.

A. Radar Information Channel Model

The radar system can be viewed within a systems model depicting the information flow through the signature sensing and processing components of a radar system as shown in Fig. 1 [19].

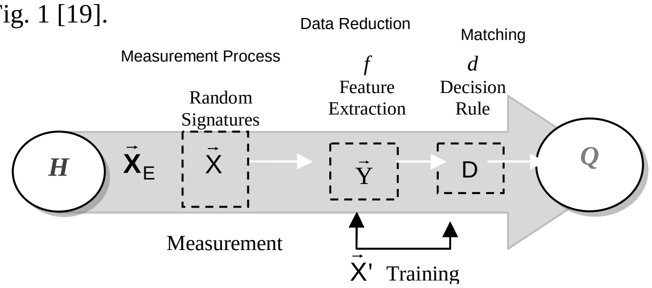

Figure 1. Information Theoretic Radar Channel Model.

The relationship between the true state (*H*) of a target under measurement and the decision state (*Q*) of a classifier algorithm is the basis chosen for performance characterization. Successful flow of information results in agreement between *H* and *Q*. This channel model differs from the communications channel model in several important respects. In the communications channel the *transmission signal is designed* to maximize information flow through a *fixed channel*. In the case of the radar information channel, the problem involves the *design of the channel* for maximum information flow given a *fixed input* (the scattered field of the targets).

B. Radar Sensor Model

The use of high range resolution (HRR) radar measurements has been useful in the support of research of signature exploitation capability within airborne platforms. In view of the uncertainties such as those listed in Table I, the HRR signature may be considered to be a random vector. Given the dynamic nature of the phenomenon underlying these uncertainties the statistics associated with the HRR random vector are often time varying. Therefore, the measured HRR signature of the target at a given time is a realization of a multidimensional random process (time varying random vector). If the target statistics are assumed to be stationary (constant with time), the sample signatures associated with this random vector correspond to a stationary random process that can be modeled and studied. For the purposes of this study, uncertainty due to thermal noise, target aspect, and training limitations will be studied.

978-1-4244-5813-4/10/$26.00 ©2010 IEEE

C. *Target Scattering Model*

In the high frequency regime used to obtain HRR signatures, the target may be approximated[1] as a collection of scattering centers valid over a limited aspect window and frequency band. These scattering centers may be considered to be localized to a point and represent a variety of scattering phenomena ranging from specular reflection to diffraction phenomena such as edge and tip diffraction. The mathematical definition of the HRR signature is developed from the normalized scattered field in (1) for a single frequency. In equation (1) σ is the radar cross section and *R* is the range to the scattering center. $\vec{E}^s$ and $\vec{E}^i$ are the scattered field and the incident field respectively.

$$\sigma = \lim_{R\to\infty} 4\pi R^2 \frac{\left|\vec{E}^s\right|^2}{\left|\vec{E}^i\right|^2} \tag{1}$$

Using scattering center modeling and the far field approximation, equation (1) can be written in terms of the target aspect angle and the transmitted wavelength as shown in equation (2) [20].

$$S(\theta,\phi,\lambda) = \sum_{m=1}^{M} \sqrt{\sigma_m}\; e^{j\frac{4\pi}{\lambda}R_m} \tag{2}$$

In equation (2) S is the band-limited frequency response of the target. Applying matched filter processing and the discrete Fourier transform, the measured HRR signature can be modeled for a given aspect angle and range of frequencies present in the transmitted waveform. The variation in signature phenomenology due to the uncertainties in the aspect angle are captured in the signal model illustrating that the HRR signature must be viewed as a random process represented here as $\vec{X}$. A small window of aspect angles, typically less than $5^\circ \times 5^\circ$ in azimuth and elevation around a specified aspect, is chosen for targets of interest at X-band frequencies (8-12 GHz) in the following development. The targets are electrically large with dimensions in range and cross-range of many wavelengths. In all cases the thermal noise is assumed to be additive.

D. *Decision Algorithm*

The algorithm used to perform the feature extraction function *f* in Fig. 1 is based on the principle components [21] (or modes) of the complex HRR signature process $\vec{X}$. The discriminant D in Fig. 1 then becomes the power associated with the projection of the sampled test signature vector from $\vec{X}$ with the principle eigenvector (or eigenvectors) associated with our best characterization of $\vec{X}$. The eigenvectors of $\vec{X}$ are estimated from the training process $\vec{X}'$ as shown in Fig. 1.

[1] Note that as an approximation, scattering centers may not be sufficient to adequately represent complex electromagnetic phenomenology: e.g., creeping waves, edge waves, and cavity scattering.

E. *Decision Rule Training*

The training process component $\vec{X}'$ in Fig. 1 represents the best possible statistical characterization of the observed signature process $\vec{X}$. Signature training processes must represent the radar measured signature process across a wide range of target articulations and configurations as well as under many operating conditions including clutter, obscuration, and other sources of RF interference. Construction of a signature training database derived entirely from measurements is expensive and can be an impractical proposition. It is possible to construct a signature database using electromagnetic scattering codes. However, given the complexity of typical targets and the challenge of modeling a variety of electromagnetic scattering phenomena ranging from specular reflection to edge diffraction, smooth surface diffraction etc., computation of signatures with sufficient accuracy is a challenging task [12]. Within this analysis the dissimilarity of $\vec{X}$ with $\vec{X}'$ will be generated using scattering center decimation. The number of scattering centers associated with peak features in $\vec{X}$ is reduced incrementally. $\vec{X}' \equiv \vec{X}$ only when $\vec{X}$ is used for the training of the decision rule *d* in Fig. 1.

F. *Numerical Experiments*

The information flow between components within the radar system is studied through several numerical experiments. The design of the sensor, decision rule training approach, and algorithm feature are varied under specified uncertainty conditions. The information loss at each component is computed for each design/uncertainty configuration. The various experimental configurations are outlined in Table II. The baseline configuration is as defined in Case 1 with the signal-to-noise ratio (SNR) set to 20 dB and the target aspect uncertainty (θ) set to $+/-2^\circ\,(3\sigma)$ [2] in both azimuth and elevation. Cases 1-4 are designed to study the sensitivity of information loss within various components of the system to changes in key design parameters and uncertainty conditions. In Case 1 the sensitivity to additive thermal noise is studied at baseline conditions. In Case 2 the sensitivity of baseline component information loss is studied as a function of increasing bandwidth. In Case 3 the number of modes used to construct the signature feature (number of eigenvectors) is varied to study the effect of information loss within components. In Case 4 baseline conditions are used to study the information loss associated with the degree of similarity between the training process $\vec{X}'$ and the measured process $\vec{X}$.

TABLE II
EXPERIMENTS

Case	**System Component Design**

[2] The target aspect angle is modeled as Gaussian in Azimuth & Elevation falling within $+/-2^\circ\,(3\sigma)$ of the mean.

	Sensor, $\vec{X}$	Feature, *f*	Training Process, $\vec{X}'$
1	Bandwidth: 500 MHz Dynamic Range: 10 Bit SNR: 0-20 dB, $\theta = 4^\circ\ 3\sigma$	Mode: 1	$\vec{X}' \equiv \vec{X}$
2	Bandwidth: 100-1000 MHz Dynamic Range: 10 Bit SNR: 20 dB, $\theta = 4^\circ\ 3\sigma$	Mode: 1	$\vec{X}' \equiv \vec{X}$
3	Bandwidth: 500 MHz Dynamic Range: 10 Bit SNR: 20 dB, $\theta = 4^\circ\ 3\sigma$	Modes: 1-15	$\vec{X}' \equiv \vec{X}$
4	Bandwidth: 500 MHz Dynamic Range: 10 Bit SNR: 0-20 dB, $\theta = 4^\circ\ 3\sigma$	Mode: 1	$\vec{X}' \neq \vec{X}$

*Note: Equiprobable Binary Hypothesis Conditions

III. THEORY

It is desired to quantify the impact of uncertainty on the information loss associated with selected components. Two theorems from information theory play key roles in our development of this relationship. The first theorem is Fano's Inequality which relates information theoretic quantities to the Probability of Error (P_e) criterion for a target classification system [22]. The second theorem is the Data Processing Inequality [22].

A. Fano's Inequality and The Data Processing Inequality

The Data Processing Inequality allows the analysis of the flow of information from the measured target returns through the signal processing architecture and into the decision rule algorithm, detailing where information is lost. In this manner, stages in the information processing pipeline where information is lost can be identified, analyzed and optimized, leading to improvement in overall system performance.

The discrete random variable $\boldsymbol{H}$ represents which of N possible hypotheses has occurred. For example, when N=2 we can have outcomes A or B. Conditioned on the generating hypothesis $\boldsymbol{H}$, there is typically a multidimensional encoded source[3] $\vec{X}_E$ which when subjected to the uncertainties associated with measurement are realized as the random radar returns from the scattering of the object under measurement. After mixing, filtering, and signal processing, these returns become the measured random signature vector $\vec{X}$. Typically the multidimensional random feature $\vec{Y}$ is extracted from $\vec{X}$ in order to support the desired function of the exploitation system. The decision rule training process $\vec{X}'$ is used to develop a full characterization of $\vec{Y}$. This characterization is applied to sample measured signatures of $\vec{X}$ to generate the random discriminant D. $\vec{X}'$ is also used in conjunction with sparse samples from $\vec{X}$ to determine the optimal algorithm decision rule *d*. The exploitation algorithm applies the decision rule *d* to D. The discrete random variable $\boldsymbol{Q}$ denotes the classifier algorithm decision of which hypothesis occurred based on the signal processed feature $\vec{Y}$ and resulting discriminant D.

Fano's equality for the model in Fig. 1 is given in (3).

$$S(P_e) = \delta - P_e \cdot \log_2(\text{N-1}) + S(\boldsymbol{H}/\boldsymbol{Q}) \tag{3}$$

In (3) P_e is the probability of error of the decision rule algorithm, S($\boldsymbol{H}$) is the Shannon entropy of the discrete random variable $\boldsymbol{H}$. S($\boldsymbol{H}/\boldsymbol{Q}$) is the conditional entropy of $\boldsymbol{H}$ given $\boldsymbol{Q}$. δ is a bias offset derived from symmetries in the data and decision algorithm [19], as well as the signal processing algorithm. Typically δ is small and to a first approximation may be neglected. This approximation will be made in the following analysis. I($\boldsymbol{H};\boldsymbol{Q}$) is the mutual information between $\boldsymbol{H}$ and $\boldsymbol{Q}$ [23]. Using I($\boldsymbol{H};\boldsymbol{Q}$)=S($\boldsymbol{H}$) – S($\boldsymbol{H}/\boldsymbol{Q}$) and (3) we get (4) below.

$$S(P_e) \approx -P_e \cdot \log_2(\text{N-1}) + S(\boldsymbol{H}) - I(\boldsymbol{H};\boldsymbol{Q}) \tag{4}$$

Equation (4) can then be written more completely for N=2 as in (5) below.

$$S(P_e) \approx S(\boldsymbol{H}) - I(\boldsymbol{H};\boldsymbol{Q}) \tag{5}$$

Equation (5) can be written in terms of the inverse function F as shown in (6).

$$P_e \approx F(S(\boldsymbol{H}) - I(\boldsymbol{H};\boldsymbol{Q})) \tag{6}$$

Assuming that the P_e lies between [0,1/2], F is a deterministic strictly monotonically increasing function that maps information theoretic quantities into a P_e. The quantity in (7) is the end to end information loss (IL) for the system.

$$\text{IL} \approx S(\boldsymbol{H}) - I(\boldsymbol{H};\boldsymbol{Q}) \tag{7}$$

Minimizing the information loss minimizes the system P_e.

The entropic quantity S($\boldsymbol{H}$) is determined by the a priori probabilities of the outcomes of the random variable $\boldsymbol{H}$, which correspond to the different target classes. Since F is a known function, the relation $P_e \approx$ F(S($\boldsymbol{H}$) – I($\boldsymbol{H};\boldsymbol{Q}$)), for fixed S($\boldsymbol{H}$), determines the mutual information I($\boldsymbol{H};\boldsymbol{Q}$) needed to achieve a specified P_e. For example, for an equiprobable binary hypothesis scenario[4], S($\boldsymbol{H}$) = 1 Bit, $P_e \approx$ F(1 – I(H;Q)). Specifying a desired P_e determines the amount of allowed IL. How the IL budget is spent as information cascades from the input collected signature space to the classifier output can be traded off. Fig. 2 is an abstract diagram indicating possible tradeoffs.

Information losses within the channel may be studied with respect to various uncertainties such as those in Table I. The Data Processing Inequality states that information can only be lost in the information channel as shown in (8).

[3] $\vec{X}_E$ in this context is deterministic. Given the "unknowable" nature of this code through measurement or modeling, the code itself is actually stochastic in nature and will be treated as such in future work.

[4] The selection of the uniform prior on $\boldsymbol{H}$ is for illustration purposes.

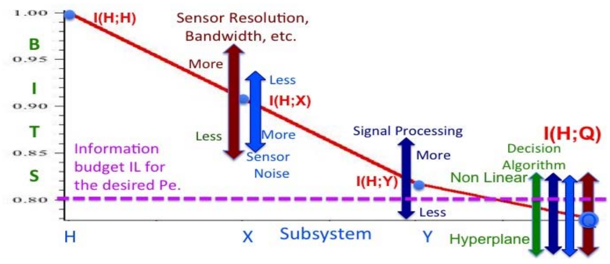

Figure 2. An abstract sketch of information flow tradeoffs [19].

$$\mathrm{I}(\boldsymbol{H};\boldsymbol{X}) \geq \mathrm{I}(\boldsymbol{H};\boldsymbol{D}) \geq \mathrm{I}(\boldsymbol{H};\boldsymbol{Q}) \tag{8}$$

Using the relationships in (4) and (8) the loss associated with each source within the channel can be characterized at certain points in the channel as shown in (9).

$$\mathrm{S}(\boldsymbol{H}) - \mathrm{I}(\boldsymbol{H};\boldsymbol{X}) \leq \mathrm{S}(\boldsymbol{H}) - \mathrm{I}(\boldsymbol{H};\boldsymbol{D}) \leq \mathrm{S}(\boldsymbol{H}) - \mathrm{I}(\boldsymbol{H};\boldsymbol{Q}) \tag{9}$$

Through the use of information theory based principles, a formal mathematical definition of sensor system uncertainty is possible. The fundamental relationship between the entropy associated with the probability of error and the MI between the random variables ***H*** and ***Q*** becomes the basis for our study of system uncertainty. From (4) we can see that all sources of uncertainty introduced in the channel will result in an increase in P_e. An increase in $\mathrm{S}(P_e)$ will result in a reduction in I(***H;Q***) and induce a loss in information flow and a degradation to the P_e.

Information is defined in terms of the mutual information between the "typical subspaces" [22] associated with the true object state ***H*** and the decision state ***Q***. Systems (and associated sub-component) designs that increase the MI between these "typical signature subspaces" increase the flow of information. "Uncertainty" is defined as an alteration to the typical signature subspaces (growth or movement) that results in a potential loss in the flow of information and an ultimate decrease in confidence in decision.

IV. Results

The results of the experiments outined in Table II are presented below.

In Fig. 3 the loss in information going from the hypothesis ***H*** to the discriminant D is captured by I(***H***;D). The information loss ranges from 1-0.12 Bits at 0 dB SNR to about 1-0.45 Bits at 20 dB SNR. Most of the information gain appears to be prior to 10 to 12 db SNR. Using equation (5) the Fano estimate of the system performance measure of P_e is observed to be equal to the actual P_e. Consistent with the trends in information loss, most of the reduction in P_e occurs prior to 10 to 12 dB SNR. In Fig. 4 the difference between measures I(***H***;D) and I(***H;Q***) show a loss of about 0.05 to 0.1 Bits of information in applying the discriminant to the algorithm decision rule.

In Fig. 4 the effects of bandwidth on the loss of information is illustrated. The trends are similar to those in Case 1 with I(***H***;D) achieving most of the loss reduction and system performance (P_e) at about 500 MHz (approximately 0.55 Bits of loss).

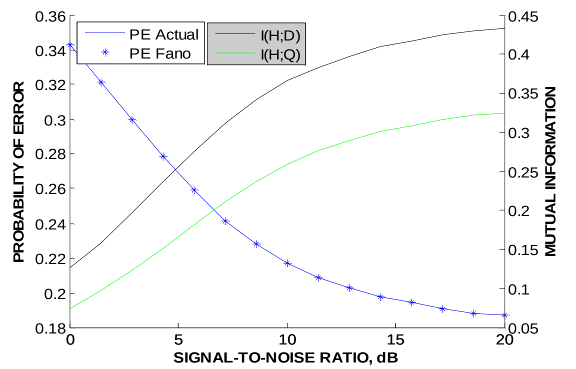

Fig. 3. Case 1: Component Information Loss and System Probability of Error as a Function of Signal-to-Noise Ratio, BW=500 MHz

The loss in information at the discriminant point D ranges from .87 Bits at 100 MHz of BW and reduces to about 0.5 Bits at 1 GHz of BW. A loss of less than 0.1 Bits of information is incurred in applying the algorithm decision rule to the discriminant (D).

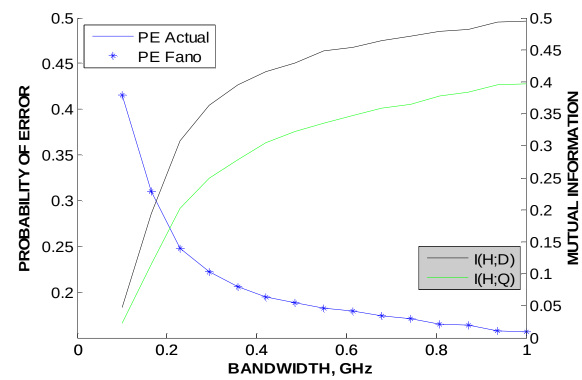

Fig. 4. Case 2: Component Information Loss and System Probability of Error as a Function of System Bandwidth, SNR=20 dB

In Case 3 the effects of increased dimensionality in the feature space $\vec{\mathrm{Y}}$ are studied for the baseline conditions. Fig. 5 illustrates the reduction in information loss as a function of an increase in the number of modes within the feature $\vec{\mathrm{Y}}$. By the time $\vec{\mathrm{Y}}$ is constructed using five modes the measure P_e reaches near full performance. In Fig. 6 the case 4 experiment results illustrate the loss of information due to limitations within the training signature process. Scattering centers associated with features are incrementally removed to drive the training process $\vec{\mathrm{X}}'$ $\vec{\mathrm{X}}$ away from the training process. Within the range of the similarity index, there clearly exists a knee in the curve between index 3 and 5.

The results for all four cases are summarized in Table II below. The loss associated with various components is tabulated as a function of the four experiments (Cases 1-4).

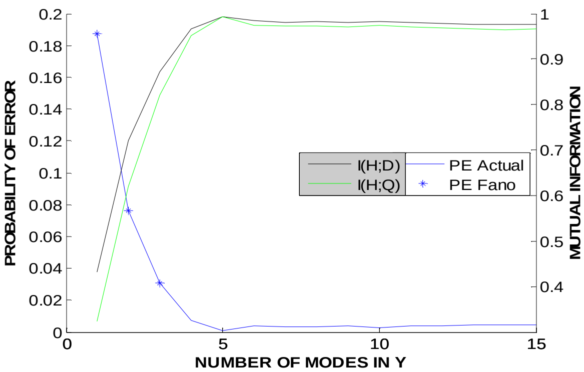

Fig. 5. Case 3: Component Information Loss and System Probability of Error as a Function of Number of Modes in Feature Y, SNR=20 dB, BW=500 MHz

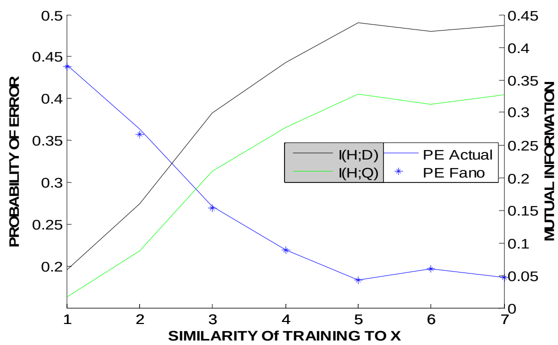

Fig. 6. Case 4: Component Information Loss and System Probability of Error as a Function of Training Process Similarity to Measured Signature Process, SNR=20 dB, BW=500 MHz

TABLE III
INFORMATION LOSS BUDGET

System Component	**Information Loss, Bits**			
	Case 1 SNR	**Case 2 BW**	**Case 3 Y**	**Case 4 ($\vec{X}'$, $\vec{X}$)**
Source-to-Measurement (X)	TBP (TBP)*	TBP (TBP)*	TBP (TBP)*	TBP (TBP)*
Measurement –to-Discriminant (D)	0.55-0.88 (0.55)*	0.45-0.95 (0.55)*	0.03 -0.1 (0.5)*	0.55-0.8
Decision Rule Application (Q)	< 0.05 (0.1)*	< 0.1 (0.1)*	< 0.05 (0.1)*	< 0.1 (0.1)

*Baseline conditions, ** TBP: To be provided in final draft

V. CONCLUSION

In general it appears that for this binary decision algorithm, several observations can be made. With 1 Bit total information in the channel, the component loss due to the uncertainty within the application of the algorithm decision rule is small compared to the loss incurred by the transformation of the encoded signature to the discriminant D. *Additional insight will be available when the values of I(**H;** $\vec{X}$)are available to provide the loss associated with the measurement of the target class hypotheses.*

The selection of 500 MHz for transmit BW and five modes within $\vec{Y}$ appear to be good design choices given the information loss curves in Figures 3-5. A SNR of at least 10 dB is required to realize optimal performance. It appears that the sensitivity to the selection of the number of modes within the feature design $\vec{Y}$ can be high when the number of modes is below a minimum cut-off. Uncertainties due to training limitations and that associated with feature design appear to be the dominant considerations within the limits of this study.

REFERENCES

[1] P.M. Woodward, I.L Davies, "A Theory of Radar Information", *Philosophical Magazine*, Vol. 41, pp. 1001-1017, Oct. 1951.

[2] P.M. Woodward, *Probability & Information Theory with Applications to Radar*, 2nd edition, Pergamon Press, 1964

[3] C. E. Shannon, "A mathematical theory of communication," *Bell System Technical Journal*, Vol. 27, pp. 379-423 and 623-656, July and Oct., 1948

[4] M. Bell, "Information Theory and Radar: Mutual Information and the Design and Analysis of Radar Waveforms and Systems", Ph.D. Dissertation, Department of Electrical Engineering, California Institute of Technology, 1988

[5] A. Leshem, O. Naparstek, A. Nehorai, "Information Theoretic Adaptive Radar Waveform Design for Multiple Extended Targets", *IEEE Transactions on Selected Topics in Signal Processing*, Vol. 1, No. 1, June 2007

[6] S. Sowelam, A. Tewfik, "Waveform Selection in Radar Target Classification" , *IEEE Transactions on Information Theory*, Vol. 46, no. 3, May 2000

[7] S. Kullback, R.A. Leibler, "On Information & Sufficiency", *Ann. Math. Stat.*, 22: pp. 79-86, 1951

[8] S. Briles, "The Theory for, and the Demonstration of, Information Theory Applied to Radar Target Identification", Ph.D. Dissertation Kansas State University, 1992

[9] A. M Home, D. Malvern "An information theory for the prediction of SAR target classification" *in Proc. for Algorithms for synthetic aperture radar imagery. Conference N°8, Orlando FL, 2001*, Vol. 4382, pp. 404-415

[10] J.C. Principe, D. Xu, Q. Zhao, J.W. Fisher "Learning form Examples with Information Theoretic Criteria", *Journal of VLSI Signal Processing 26*, 61–77, 2000

[11] J. O'Sullivan, M. DeVove, V. Kedia, M. Miller, "SAR ATR Performance Using a Conditionally Gaussian Model" *IEEE Transactions on Aerospace Electronics Systems Vol.37*, No. 1, Jan. 2001.

[12] J. Malas, K. Pasala, "Information Theory Based Signature Analysis", *In the Proc. of 2007 IEEE Aerospace Conference*, Mar. 2007

[13] E. Fishler, A. Haimovich, R. Blum, L. J. Cimini, D. Chizhik, and R. A. Valenuela, "Spatial diversity in radars-models and detection performance," *IEEE Transactions on Signal Processing*, Vol. 3, no. 54, pp. 823-838, Mar. 2006

[14] K. Forsythe and D. Bliss, "Waveform correlation and optimization issues for MIMO radar," in *Proc. 39th Asilomar Conf. Signals, Systems, and Computers*, Monterey, CA, 2005, pp. 1306-1310.

[15] D. R. Fuhrmann and G. San Antonio. "Transmit beamforming for MIMO radar systems using partial signal correlation," *IEEE Transactions Aerospace Electronics Systems* Vol. 43, no. 4, Oct. 2007.

[16] D. Bliss and K. Forsythe, "Multiple-input Multiple-output (MIMO) radar imaging: Degrees of freedom and resolution" in *Proc. 39th Asilomar Conf. Signals, Systems, and Computers*, Monterey, CA, 2003, pp. 54-59.

[17] N. Tishby, F. Pereira, and W. Bialek. "The information bottleneck method" in *Proc. 37th Allerton Conf. on Communication and Computation*, 1999.

[18] J. Mandel, *The Statistical Analysis of Experimental Data*, National Bureau of Standards, Washington, *D.C.*, Dover Publications Inc.

[19] J. A. Cortese, "Information Theory and Radar Signal Processing", Air Force Research Lab, Georgia Institute of Technology College of Computing, Tech. Rep. Mar., 2009, Unpublished

[20] N. Levanon, *Radar Principles*, John Wiley & Sons, 1988

[21] K. Fukunaga, *Statistical Pattern Recognition*, Academic Press, 1990

[22] T. Cover, J. Thomas, *Elements of Information Theory*, New York: Wiley, 1991

[23] This paper cleared for public release: 24-FEB-10 88, #ABW-10-0806